Super-diffusion and MHD shocks

Numerical investigation of the role of super-diffusion in shock-acceleration

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In Short

- Fluid simulation of the evolution of a magnetohydrodynamic shock
- In a turbulent medium
- Results from fluid simulations are used to trace the movement of individual particles
- In order to study the process of shock acceleration

Any disturbance at a specific point in a fluid will propagate to other parts of the fluid (e.g., ripples on a pond). As a result, the various fluid quantities (e.g., velocity, density, and magnetic field strength) at a given location will change when the disturbance reaches this point in space. If the disturbance is propagating faster than the speed of sound, it will lead to the formation of a shock, and it is possible to divide the fluid into a region that has been affected by the passing of the shock (downstream), and a region that has not been affected (upstream). One characteristic of a shock is that the fluid quantities change rapidly, almost discontinuously, between the upstream and downstream regions, as illustrated in Figure 1.

![Figure 1](image1.png)

**Figure 1:** Graph showing the discontinuous change in density and velocity due to the presence of a shock at 0.85. The downstream and upstream regions are to the left and right of the shock, respectively. The graph is for fluid simulations of the well-known Sod shock-tube problem.

While not common in everyday life on earth, shocks are ubiquitous in the universe. For example, stars that are roughly 10 times heavier than the Sun will end their lives in a cataclysmic explosion, known as a supernova explosion. As a results, stellar material is driven into the interstellar medium at supersonic speeds, leading to the formation of a shock (see e.g., [1] for a review).

To understand the influence of astrophysical shocks, it is important to note that interstellar space is not empty, but is filled with a very low density "gas" of electrically charged particles and magnetic fields, i.e., a plasma. An immediate consequence of the shocks is that it will lead to the formation of turbulence in this plasma. As a result, the magnetic fields in the plasma are not smooth, but change direction randomly from point to point (although it is possible for the field, on average, to point in a certain direction). Due to the fact that a magnetic field exerts a force on a charged particle, the random nature of the interstellar magnetic field means that the particles in the plasma will feel a continually changing force. The particles will therefore not move in a straight line, but will rather exhibit random-walk motion (also known as diffusion), as illustrated in Figure 2.

![Figure 2](image2.png)

**Figure 2:** Numerical simulation of a turbulent magnetic field. The colours indicate the strength, and the arrows the direction of the magnetic field. The blue line is an example of the random-walk (diffusive) motion of a particle.

A second, but equally important, consequence of astrophysical shocks is that they can accelerate electrically charged particles to very high energies (see e.g., [2] for a review). When a particle crosses the shock the sudden difference in velocity and magnetic field strength causes the particle to gain energy. However, in order for the shock acceleration mechanism to work efficiently, a particle must cross the shock multiple times. In the interstellar medium multiple crossings are only possible due to the fact that the particles undergo diffusive motion (see Figure
Figure 3: Simplified illustration of the trajectory of a particle near a shock. Note that the velocity of the fluid is represented by the length of the arrows, i.e., the velocity to the left of the shock is larger. The particle gains energy every time it crosses the shock, regardless of the direction that the particle is moving in.

As diffusion plays a critical role in shock acceleration, it is important to understand this process in detail. While a fundamental theory of diffusion is still being developed, it is nevertheless possible to describe the process using a simple equation. If $\Delta x$ represents the distance between the initial and final position of a particle at a time $t$, then diffusion is described by

$$\langle \Delta x^2 \rangle \propto t^\beta,$$

where the brackets $\langle \rangle$ denote the average taken over the trajectories of a large number of particles. In general, it is assumed that $\beta = 1$. However, recent numerical simulations [3] show that for particles propagating in the interstellar medium one has $\beta = 3$, with any scenario where $\beta > 1$ defined as super-diffusion.

While previous research into shock acceleration has generally assumed the value of $\beta = 1$, an analytical investigation by [4] used the more correct value of $\beta = 3$, with the authors finding that this significantly influences the shock acceleration process.

Due to the complex nature of the problem, analytical investigations are often limited to simplified scenarios. Thus, in order to obtain a better understanding of the shock acceleration process one must necessarily use numerical simulations. The main aim of the project is to test the prediction of [4] and to obtain an quantitative understanding of the acceleration process by making use of high resolution numerical simulations.

The simulations are based on solving the a set of equations that describe the conservation of mass, momentum, energy, and magnetic flux in a fluid, along with the equation that describes the motion of a charged particle in a magnetic field. By tracing the trajectories of a large number of particles in a shocked, turbulent fluid, one will be able to better understand the shock acceleration process.

The proposed project is important for two reasons. Firstly, recent observations of supernova shocks have revealed evidence for super-diffusion [5]. Secondly, it is generally believed that shock acceleration is responsible for producing the high-energy particles, known as cosmic rays, that are observed on earth and by experiments in space. While a fraction of cosmic rays are produced in our own solar system, the majority of observed cosmic rays are produced in our Galaxy, as well as other galaxies. The cosmic rays have energies that range from $10^8$ eV to $10^{20}$ eV, with the upper limit being roughly 10 million times higher than the energy of the particles produced at the most powerful particle accelerator, the Large Hadron Collider (see e.g., [6] for a review on cosmic rays).

Understanding shock acceleration is thus crucial for understanding an important part of the universe.

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http://www.uni-potsdam.de/astroparticle/plasmaastrophysik.html

More Information


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