Elastic Turbulence in 3D geometries.

Characterising elastic turbulence in viscoelastic fluids at low Reynolds numbers.

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In Short

• Elastic turbulence, observed in viscoelastic fluids, enhances heat and mass transport at very low Reynolds numbers.

• Beyond a critical Weissenberg number, an elastic instability causes a supercritical transition from a laminar to a turbulent flow.

• The onset of elastic turbulence in a three-dimensional geometry using numerical solutions of the Oldroyd-B model will be investigated.

Viscoelastic fluids, such as polymeric fluids, have exceptionally useful properties compared to Newtonian fluids. At small scales, viscoelastic fluids exhibit transitions from laminar flows to non-laminar, chaotic and turbulent flows. This is due to elastic turbulence [1], which bears similar qualities as inertial turbulence. This property is useful for heat and mass transport in flows at the micron scale, where in Newtonian fluids transport is dominated by diffusion.

The relevant dimensionless number characterizing viscoelastic fluids is the Weissenberg number $Wi$, which compares the polymer relaxation time to the characteristic time of the flow dynamics.

Elastic turbulence (see Fig. 1) shows many characteristics similar to inertial turbulence [2]. The fluid flow exhibits a significant increase in flow resistance [3], random fluctuations of fluid velocity that increase with fluid elasticity, intensified mixing of mass, and a broad range of activated temporal or spatial frequencies with power-law scaling [5]. The elastic component of the fluid, quantified by the elastic stress tensor $\tau$, is affected by two processes: stretching of the polymer molecules by velocity gradients and relaxation of elastic stresses. The dominant process depends on the value of the Weissenberg number. At $Wi \ll 1$ the relaxation of the polymers is much faster than the stretching time due to velocity gradients. The polymer acts like a rigid rod and the fluid flow is Newtonian. For $Wi > 1$ stretching due to velocity gradients overcomes relaxation. Polymers are considerably elongated [3] and the polymeric stresses grow. This effect is further enhanced by curved streamlines [6,7].

Recently, we investigated the occurrence of elastic turbulence of a viscoelastic fluid in a 2D Taylor-Couette geometry using numerical solutions of the Oldroyd-B model. In the Taylor-Couette geometry the viscoelastic fluid is contained between two concentric cylinders. In our case the outer cylinder rotates with constant angular velocity while the inner one is fixed. We observed a supercritical transition from the (purely azimuthal) laminar Taylor-Couette flow to a flow profile with an additional secondary, turbulent flow [4]. In Fig. 2, snapshots of the radial component of the velocity field are presented indicating the emergence of a (radial) secondary flow. The secondary flow is caused by an elastic instability beyond a critical Weissenberg number. The order parameter, the time average of the secondary-flow strength, follows the scaling law $\Phi \propto (Wi - Wi_c)^\gamma$ with $Wi_c = 10$ and $\gamma = 0.45$ (see Fig. 3). Additionally, the flow resistance increases beyond $Wi_c$.

The temporal power spectra of the velocity fluctuations show a power-law decay with a characteristic exponent in the range $2 < \alpha < 4$. The characteristic exponent $\beta$ for the spatial power spectra of the velocity fluctuations obeys the necessary condition $\beta > 3$, associated with elastic turbulence [7], for all $Wi > Wi_c$.

In our work, the Navier-Stokes equations and the Oldroyd model are solved up to large Weissenberg numbers using the program OpenFOAM® [8], which is an open-source finite-volume solver for computational fluid dynamics simulations on polyhedral grids. In particular, it can handle the high-Weissenberg-number problem, where numerical instabilities occur.

![Figure 1: Snapshots of the turbulent flow (experiment) as well as stress and velocity fields from numerical solutions of the Oldroyd-B model.](image1)

![Figure 2: Snapshots of the normalised radial velocity component $u_r/u_{max}$ for Weissenberg numbers of A) $Wi = 12.6$, B) $Wi = 50.3$ and C) $Wi = 106.8$ in a 2D Taylor-Couette geometry.](image2)
at high $Wi$. We adopt a specialised solver for viscoelastic flows implemented in OpenFOAM®: called rheoTool, which is based on a solver for complex fluids [9]. It has been shown to have second-order accuracy in space and time [10]. We work at a very low Reynolds number appropriate for flows at the micron scale and keep its value fixed during all simulations. Thus, the observed turbulence is only due to the polymeric, i.e., the elastic component of the fluid.

By restricting our investigations to two spatial dimensions, we have the simplest implementation of a Taylor-Couette flow. In the simulations we use a mesh refinement towards the inner cylinder, where velocity gradients become larger. The mesh refinement is such that the ratio of the radial grid size at the inner cylinder to the one at the outer cylinder is 10. However, the use of a 2D geometry is a simplification. It cannot reproduce the highly three-dimensional structure of the experimentally observed elastic turbulence and its experimental realisation is questionable.

In this project we plan to expand our analysis of elastic turbulence in viscoelastic fluids to 3D geometries using the same approach as for the 2D case described above. OpenFOAM® is perfectly suited to perform parallel simulations. To handle the significant demand of performing computational fluid dynamics on viscoelastic fluids in three dimensions, access to a fast parallel computing facility is required. Preliminary results on a 3D parallel plate geometry demonstrate impressive qualitative similarities between the numerical solutions of the Oldroyd-B model and experimental results. In Fig. 1 snapshots of the magnitude of the stress tensor and the magnitude of the velocity field are plotted, as well as a snapshot of experimental observations on the same geometry. In order to calculate the power spectra of the velocity fluctuations and compare them to experimental results, longer time series and better statistics are required. By optimally using the resources provided in this project, we will investigate the characteristics of elastic turbulence: power spectra of velocity fluctuations, flow resistance and the order-parameter as a function of the Weissenberg number. We will concentrate on two exemplary 3D geometries: the van Karman swirling flow in a parallel-plate geometry and the Taylor-Couette geometry.

Figure 3: Elastic instability: Order parameter $\Phi$ plotted versus Weissenberg number $Wi$. The dotted line shows the scaling law $(Wi - Wi_c)^\gamma$ with $Wi_c = 10$ and $\gamma = 0.45$. The positions A, B, and C correspond to the snapshots of fig. 2. Upper inset: Azimuthal stress component $\Gamma$ at the outer cylinder as a measure for the flow resistance. Lower inset: Normal stress difference $\Sigma$ and its value for the base flow (dotted line).

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More Information


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