Convection in rocky planets: how well can the mantle be stirred?

3D Convective Mixing in Planetary Interiors with Strongly Variable Rheological Properties
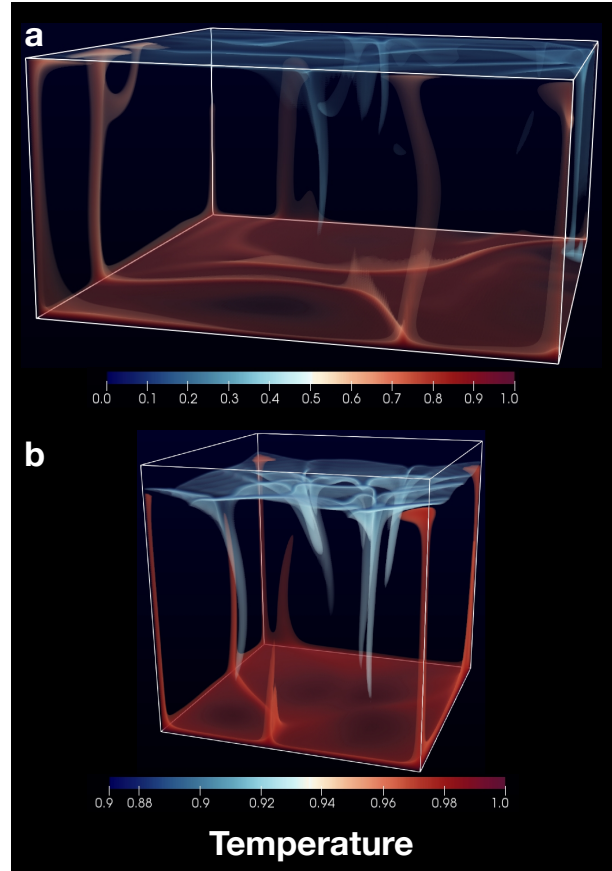
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In Short

- The rocky mantles of the Earth and terrestrial planets undergo solid-state convection

- Isotopic analyses of surface rocks and meteorites suggest the existence of large-scale compositional reservoirs in the deep interior of terrestrial planets that survived convective stirring and mixing for billions of years

- The characterization of mixing is computationally challenging and so far the problem has been addressed at realistic conditions only with 2D simulations

- We carried out simulations of 3D thermal convection both with constant and strongly temperature-dependent viscosity

- Compositional mixing in 3D convection exhibits a complex behaviour, with important differences with respect to 2D

Figure 1: *(a) Iso-surfaces of the non-dimensional temperature field for a simulation with constant viscosity and an aspect-ratio two and Rayleigh number of 10^8, and (b) with temperature-dependent viscosity with aspect-ratio one and reference Rayleigh number of 10^9 (computed at the bottom of the domain). Red and blue surfaces indicate up- and downwellings. For temperature-dependent viscosity, convection takes place beneath an insulating stagnant lid.*

During the first accounting period, we used our finite volume code Gaia [1] to carry out simulations of 3D thermal convection both with constant and strongly temperature-dependent viscosity aimed at characterizing mantle mixing. A constant viscosity leads to a mobile surface. This represents a reasonable first-order approximation for convection in the Earth, which features plate tectonics and hence a mobile surface (Figure 1a). A temperature-dependent viscosity leads instead to the formation of a stagnant-lid, i.e. an immobile surface as for Mars, Mercury, the Moon and, presently, Venus. The stagnant lid does not deform and convection takes place beneath it (Figure 1b).

An effective way to quantify the mixing properties of convective systems is the computation of the stirring efficiency as measured by the maximum finite-time Lyapunov exponent (λ_{max}), which describes Lagrangian deformation [2]. Establishing how λ_{max} scales with the vigor of convection as measured by the Rayleigh number (i.e. the non dimensional number describing the ratio of buoyancy to viscous forces) is of fundamental importance to characterize mantle mixing through the evolution of planetary interiors. Studies based on 2D convection simulations

have shown that λ_{max} scales with the Rayleigh number the same way as the root-mean-square (RMS) velocity (v_{RMS}) [2,3]. The latter follows a power law scaling of the kind $v_{RMS} \sim Ra^\gamma$, where γ is an exponent whose asymptotic value predicted by boundary layer theory is 2/3. The Lyapunov exponent is thus expected also to scale as Ra^γ, i.e. with the same exponent of v_{RMS}, which suggests that the velocity field is already a good indicator of the mixing efficiency of convection.

For the isoviscous and variable-viscosity simulations, Figure 2 shows scaling laws for v_{RMS} and λ_{max} as a function of the Rayleigh number for 3D convection with constant (Figure 2a and 2b) and variable viscosity (Figure 2c and 2d). In both cases we found that v_{RMS} scales with an exponent γ close to the the-

Constant viscosity, AR=2 **Variable viscosity, AR=1**
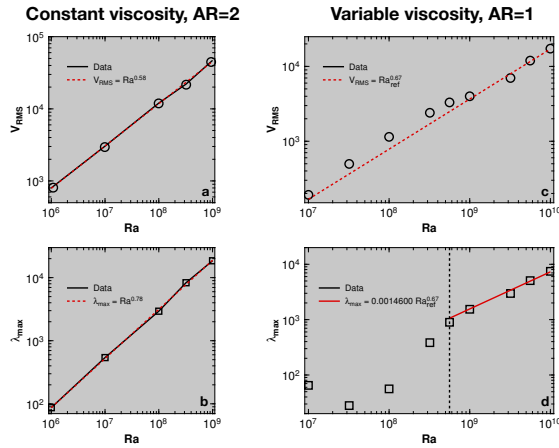
Figure 2: *Scaling of RMS velocity v_{RMS} (top line) and finite-time Lyapunov exponent λ_{max} (bottom line) with the Rayleigh number for 3D simulations with constant (left column) and variable (right column) viscosity.*

oretical one, namely $\gamma = 0.58$ for the isoviscous case and $\gamma = 0.67$ for the variable viscosity case. However, λ_{max} exhibits a complex and unexpected behaviour. For isoviscous cases, rather than a scaling with the same exponent γ as known from previous findings based on 2D simulations, we obtained for λ_{max} a significantly higher exponent ($\gamma = 0.78$). The physical reason for this increased stirring efficiency is presently still unclear but likely due to 3D effects, which we are currently investigating. For cases with temperature-dependent viscosity, we identified two new, qualitatively different regimes. For (reference) Rayleigh numbers lower than a certain threshold of about $5 \cdot 10^8$, mixing is heterogeneous with different regions of the domain undergoing different degrees of deformation. By contrast, for Rayleigh numbers higher than the above threshold, mixing is again homogeneous, with λ_{max} following a scaling law with an exponent γ equal to the one with which the RMS velocity scales.

Over the new accounting period, we will carry out additional simulations that will allow us to verify the robustness of the above results as well as new sets of runs to describe convective mixing in the framework of more complex and realistic flows. In particular, we will assess the influence on mixing of different degrees of temperature dependence of the viscosity, as well as the influence of the additional dependence of the viscosity on pressure and strain rate.

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More Information

[1] C. Hüttig, N. Tosi, W.B. Moore, An improved formulation of the incompressible Navier-Stokes equations with variable viscosity, Phys. Earth. Planet. Inter., 220, 11–18 (2013).

[2] Samuel, H., V. Aleksandrov, and B. Deo, The effect of continents on mantle convective stirring, Geophys.Res. Lett.,38(4), 2011

[3] H. Samuel and N. Tosi, The influence of post-perovskite strength on the Earth's mantle thermal and chemical evolution, Earth. Planet. Sci. Lett., 323, 50–59 (2012).