The butterfly effect in turbulence

Lyapunov exponents and turbulent transport in hydrodynamic and magnetohydrodynamic turbulence

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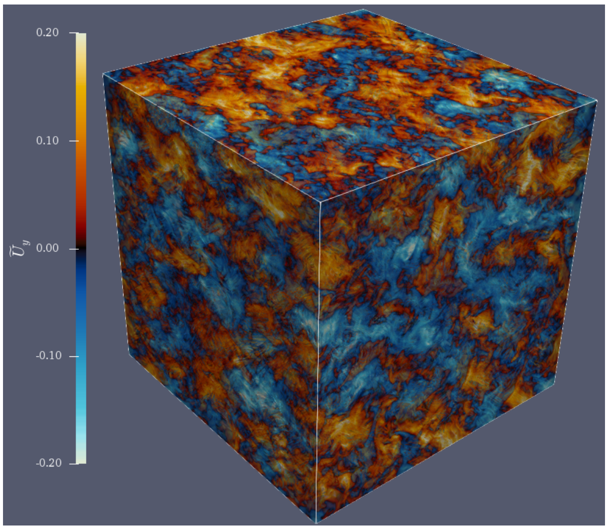

***Figure 1:** Streamwise velocity U_y in a turbulence simulation with imposed shear flow. Adapted from [3].*

In Short

- Most astrophysical flows are highly turbulent and characterized by rapidly varying multiscale motions.
- Turbulent flows are also almost always chaotic such that small changes in the initial conditions lead to exponential divergence of solutions. This is often referred to as the butterfly effect.
- The timescales describing such diverging solutions are quantified by Lyapunov exponents. The analysis of the Lyapunov exponents is required for a more fundamental theoretical understanding of turbulence.
- Turbulence is not only efficient at smoothing large-scale structures but also in building them up. Such turbulent diffusion is important in many astrophysical phenomena, such as the magnetism of the Sun.
- We will compute Lyapunov exponents and turbulent diffusion of flows and temperature from state-of-the-art simulations of turbulence and convection with and without magnetic fields.

Introduction. Almost all of the flows in the Earth's atmosphere, the interplanetary space, solar and stellar convection zones, and in other astrophysical systems are turbulent. This means that the flows change rapidly and in chaotic fashion on a huge number of scales from the system size to microscopic scales. It is important to understand the dynamics of turbulence because such turbulent flows play important roles in numerous contexts in engineering, the climate of the Earth, and the interiors of stars.
Research questions. We will measure the Lyapunov exponents of the velocity field from high resolution simulations of turbulence reaching significantly more turbulent parameter regimes than previously in the literature. Such simulations are needed to shed light on fundamental questions regarding turbulent flows and their theoretical interpretation. A number of theoretical ideas were recently presented in a theoretical study [1] which forms the basis for the numerical simulations. This is done by following the trajectories of fluid elements by means of immersed tracer particles.

Furthermore, we measure the turbulent diffusion of velocity and temperature (see first results in [2]) from three dimensional simulations of turbulence and convection relevant to stars. This part of the project is a follow-up to [3] where turbulent viscosity and magnetic diffusion where measured. Of particular interest in the current project are the effects of magnetic magnetic fields on the turbulent diffusion which has implications for various phenomena in stellar convection zones. We extend these studies to convection to address a current debate on the efficiency of turbulent diffusion in stars.
Models and methods. We use the Pencil Code [4,5] which is a free (licensed under GNU GPL v3) simulation framework for solving a great variety of physical problems ranging from gravitational waves, chemistry, planet formation to stellar magnetism. We use a simplified cubic geometry where turbulence is driven by external forcing. This approach allows the minimal ingredients necessary to the problem at hand to be included with maximal control. Figure [1] shows a representative example of the turbulence simulations.

WWW

https://www.uni-goettingen.de/en/203293.html

More Information

[1] I. Fouxon, J. Feinberg, P.J. Käpylä, M. Mond,

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[2] P.J. Käpylä, N.K. Singh, *JFM Rapids* (submitted), arXiv:2207.10335 (2022).

[3] P.J. Käpylä, M. Rheinhardt, A. Brandenburg, M.J. Käpylä, *Astron. Astrophys.* **636**, 93 (2020). doi:10.1051/0004-6361/201935012

[4] The Pencil Code Collaboration, *Journal of Open Source Software* **6**, 2807 (2021). doi: 10.21105/joss.02807

[5] Github: https://github.com/pencil-code Nordita: http://pencil-code.nordita.org/

Project Partners

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DFG Subject Area

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