# Heavy and strong: decoupling of heavy quarks in the strong coupling

## Gradient Flow coupling in a massive scheme

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### In Short

- strong coupling of quarks and gluons
- decoupling of heavy quarks at low energy
- finite-volume massive coupling based on the Gradient Flow
- simulations of Lattice Quantum Chromodynamics on fine lattices

Quantum Chromodynamics (QCD) is the theory which describes the strong interactions among the elementary particles called quarks and gluons, which are the building blocks of hadrons such as the proton and the neutron. Through the formulation of QCD on a Euclidean lattice the theory becomes amenable to computer simulations and it is possible to compute hadron properties from first principles. The running coupling of the strong force can also be computed from lattice QCD. A precise knowledge of this fundamental parameter of the standard model of elementary particles is crucial for the search of new physics as discrepancies between the model's predictions and experiments at colliders like the LHC are potentially tiny. At the moment determinations of the strong coupling from lattice QCD based on finite volume renormalization schemes yield the most precise results [1]. The effects of the charm and bottom quarks are included using the perturbative decoupling relations for heavy quarks.

In a previous project the decoupling of heavy quarks at low energies was studied in a non-perturbative setting through simulations in large volume of a model, $N_{\mathrm{f}} = 2$ QCD with two mass-degenerate heavy quarks [2]. The masses of the quarks ranged up to the charm quark mass. The decoupling of heavy quarks in a few low energy observables was investigated. The theory of decoupling applies also to couplings at low renormalization scales. This opens up a new possibility for a high precision determination of the strong coupling of QCD which we pursue in this project.

The expensive part of the computation in Ref. [1] was the calculation of the non-perturbative running of the coupling in $N_{\mathrm{f}} = 3$ QCD from low to high energies. The running is necessary because it connects the low energy hadronic regime of QCD, where the scale of the calculation is set in physical units, to the high energy perturbative regime. The contact with perturbation theory allows to include the charm and bottom quarks and express the strong coupling in the conventional $\overline{MS}$ scheme where it can be compared to other determinations. The new strategy of this project works in the following way. First a massive renormalized coupling is computed for different values of the quark mass but at the same renormalization scale $\mu_{\mathrm{dec}}$ in a theory with $N_{\mathrm{f}} = 3$ heavy quarks. We use a finite volume Gradient Flow (GF) coupling $\bar{g}^2_{\mathrm{GF}}$ [3] with Schrödinger Functional boundary conditions. The renormalization scale is given by the inverse box size $\mu_{\mathrm{dec}} = L^{-1}$. By the theory of decoupling, the massive coupling is the same as in the pure Yang–Mills theory up to corrections proportional to $M^{-1}$ (in infinite volume they are $M^{-2}$). This defines the starting point for the computation of the running in the Yang–Mills ($N_{\mathrm{f}} = 0$) theory, where a high precision can be attained, [4] but at a much lower cost than in the $N_{\mathrm{f}} = 3$ theory. The results of the pure gauge calculation is converted back to the $N_{\mathrm{f}} = 3$ theory by the use of perturbative decoupling which is very accurate at large values of $M$ [2]. In the first application period we demonstrated that the new strategy works [5], followed by a precise computation finite volume coupling in a setting with $N_{\mathrm{f}} = 3$ mass-degenerate heavy quarks for values of the quark mass ranging from charm to bottom [6]. The simulations have to follow so-called lines of constant physics which give the bare parameters of the discretized theory such that $\bar{g}^2_{\mathrm{GF}}(\mu_{\mathrm{dec}}, 0) = 3.95$, with $M/\mu_{\mathrm{dec}} \equiv z \in \{2, 4, 6, 8, 10, 12\}$ for various resolutions $a/L = a\mu_{\mathrm{dec}}$. The chosen resolutions are $L/a = 12, 16, 20, 24, 32, 40$ and $48$. For the massive coupling, fixed values of $z = ML$ determine $aM$ and therefore the hopping parameter $\kappa$. Their relation is provided by the following renormalization

$$M = \frac{M}{\overline{m}(\mu)} \frac{Z_A(\tilde{g}_0)}{Z_P(\tilde{g}_0, \mu)} m_{\mathrm{PCAC}} \left(1 + (b_A - b_P) a m_q\right),$$

with $m_{\mathrm{PCAC}} = \hat{Z}(\tilde{g}_0) m_q (1 + \hat{b}\, a m_q)$, where $a m_q = 1/(2\kappa) - 1/(2\kappa_{\mathrm{crit}})$ is the bare subtracted quark mass. The parameters in the relation between PCAC mass and RGI quark mass are known and we have carried out massless MC simulations to determine $\hat{Z}$ and $\hat{b}$. For our O($a$)-improved Wilson fermions fixed lattice spacing corresponds to fixed *improved* bare coupling $\tilde{g}_0^2$. The simulation parameter $\beta$ of the massive

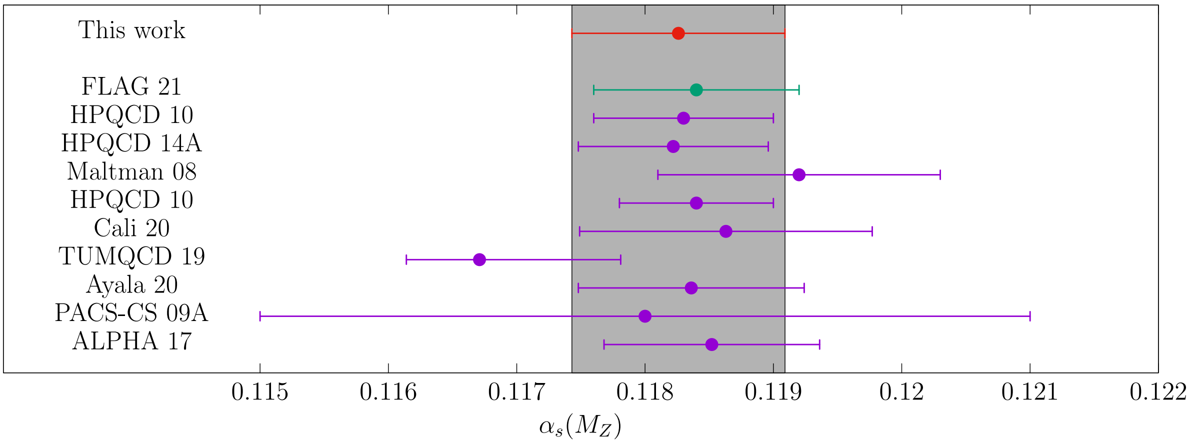


***Figure 1:*** *Our result compared with other lattice computations that enter in the FLAG average [7] (acronyms are taken from [7]).*

simulation is thus obtained from

$$\beta = \frac{6(1 + b_g\, am_q)}{\tilde{g}_0^2},$$

where the values of $\tilde{g}_0^2 = 6/\beta_{\rm LCP}$ are taken from our line of constant physics, since at zero mass the improved and unimproved couplings coincide. For $b_g$ as well as for all other improvement coefficients that are known only perturbatively, we use 1-loop values and treat the difference between tree-level and 1-loop as uncertainties, the largest effect arises from $b_g$. Once the massive couplings at a mass given by $z$ are known in the continuum limit, by using the decoupling relation as explained above, the $N_f = 3$ $\Lambda$-parameter is obtained. The latter is the fundamental QCD scale which is immediately related to the strong coupling at large values of $\mu$. The error of $b_g$ propagates to our final value of the $N_f = 3$ $\Lambda$-parameter in physical units [6],

$$\Lambda^{(3)}_{\overline{\rm MS}} = 337(10)(5)_{b_g}(3)_{M\to\infty}\,\text{MeV}\,.$$

Here the first error is statistical, the second is due to $b_g$ and the third results from the estimated uncertainty in the $M \to \infty$-extrapolation. In Fig. 1 we show a comparison of our result with other lattice computations that enter the FLAG average [7]. Our result is in good agreement with the FLAG average, our previous determination of the strong coupling [1] (ALPHA 17), and the other lattice works that enter in the FLAG average.

In the extension of the project we planned to compute the improvement coefficient $b_g$ non-perturbatively in order to reduce our final error significantly. The other sources of error are taken care of in different projects. We showed that our strategy to compute $b_g$ works and want to finish the computation in the next project period.

## WWW

https://www.physik.hu-berlin.de

## More Information

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## Project Partners

Bergische Universität Wuppertal, CERN, DESY, Trinity College Dublin