Heavy and strong: decoupling of heavy quarks in the strong coupling

Gradient Flow coupling in a massive scheme

M. Dalla Brida, R. Höllwieser, F. Knechtli, T. Korzec, A. Ramos, R. Sommer, *Institut für Physik, Humboldt Universität zu Berlin*

In Short

- strong coupling of quarks and gluons
- decoupling of heavy quarks at low energy
- finite-volume massive coupling based on the Gradient Flow
- simulations of Lattice Quantum Chromodynamics on fine lattices

Quantum Chromodynamics (QCD) is the theory which describes the strong interactions among the elementary particles called quarks and gluons, which are the building blocks of hadrons such as the proton and the neutron. Through the formulation of QCD on a Euclidean lattice the theory becomes amenable to computer simulations and it is possible to compute hadron properties from first principles. The running coupling of the strong force can also be computed from lattice QCD. A precise knowledge of this fundamental parameter of the standard model of elementary particles is crucial for the search of new physics as discrepancies between the model's predictions and experiments at colliders like the LHC are potentially tiny. At the moment determinations of the strong coupling from lattice QCD based on finite volume renormalization schemes yield the most precise results [1]. The effects of the charm and bottom quarks are included using the perturbative decoupling relations for heavy quarks.

In a previous series of works [2–4] the decoupling of heavy quarks at low energies was studied in a non-perturbative setting through simulations in large volume of a model, $N_\mathrm{f} = 2$ QCD with two mass-degenerate heavy quarks. The masses of the quarks ranged up to the charm quark mass. The decoupling of heavy quarks in a few low energy observables was investigated. The theory of decoupling applies also to couplings at low renormalization scales. This opens up a new possibility for a high precision determination of the strong coupling of QCD which we pursue in this project.

The expensive part of the computation in Ref. [1] was the calculation of the non-perturbative running of the coupling in $N_\mathrm{f} = 3$ QCD from low to high energies. The running is necessary because it connects the low energy hadronic regime of QCD, where the scale of the calculation is set in physical units, to the high energy perturbative regime. The contact with perturbation theory allows to include the charm and bottom quarks and express the strong coupling in the conventional $\overline{\mathrm{MS}}$ scheme where it can be compared to other determinations. The new strategy of this project works in the following way. First a massive renormalized coupling is computed for different values of the quark mass but at the same renormalization scale μ_dec in a theory with $N_\mathrm{f} = 3$ heavy quarks. We use a finite volume Gradient Flow (GF) coupling $\bar{g}_\mathrm{GF}$ [5] with Schrödinger Functional boundary conditions. The renormalization scale is given by the inverse box size $\mu_\mathrm{dec} = L^{-1}$. By the theory of decoupling, the massive coupling is the same as in the pure Yang–Mills theory up to corrections proportional to M^{-1} (in infinite volume they are M^{-2}). This defines the starting point for the computation of the running in the Yang–Mills ($N_\mathrm{f} = 0$) theory, where a high precision can be attained [6] but at a much lower cost than in the $N_\mathrm{f} = 3$ theory. The results of the pure gauge calculation is converted back to the $N_\mathrm{f} = 3$ theory by the use of perturbative decoupling which is very accurate at large enough values of M [4].

In the first application period we demonstrated that the new strategy works [9]. In order to tune the quark mass parameter we have determined the renormalization and improvement coefficients for the quark mass in $N_\mathrm{f} = 3$ QCD which were previously not known. We have simulated four instead of two mass points, *i.e.* $z = LM = 1.97,\ 4,\ 6$ and 8. The influence of the residual $O(a)$ boundary lattice artifacts, where a is the lattice spacing was reduced by choosing a setup where the time extent is twice the spatial one, $T = 2L$. At the same time this means that the effects of $O(1/M)$ power corrections in the decoupling relations are suppressed. We have performed these simulations on $L/a = 12,\ 16,\ 20,\ 24$ and 32 lattices. The results for the massive coupling $\bar{g}_\mathrm{GF}$ are shown in Fig. 1. The lines are continuum extrapolations at fixed values of z using two mass cuts $(aM)^2 < 1/8,\ 1/4$. The results are compatible. Clearly the continuum extrapolations become more challenging at large values of z.

Once the massive couplings at a mass given by z are known in the continuum limit, by using the decoupling relation as explained above the $N_\mathrm{f} = 3$ Λ-parameter is obtained. The latter is the fundamental QCD scale which is immediately related to the

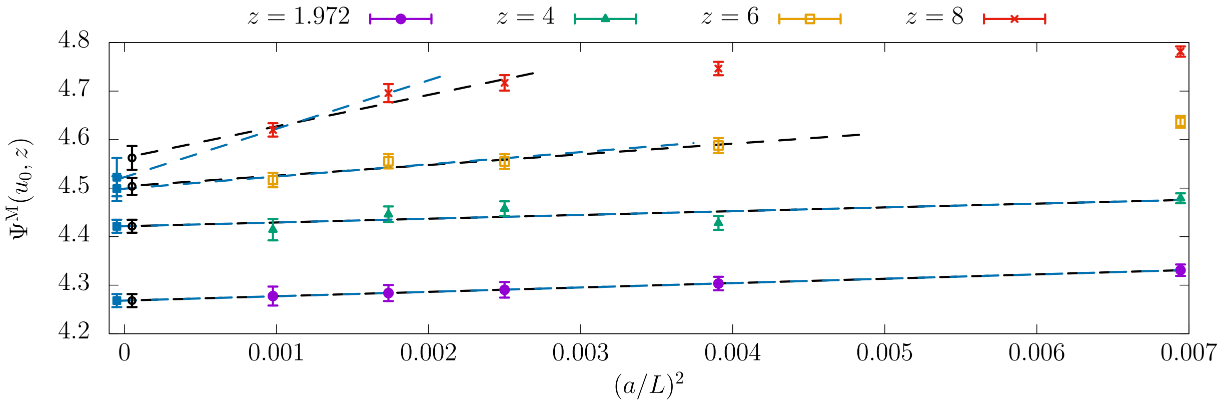

Figure 1: *Continuum extrapolations of the massive coupling $\bar{g}^2_{N_f=3}(\mu_{dec}, M)$. Here $\mu_{dec} = 1/L$ is the inverse box size and $z = ML$.*

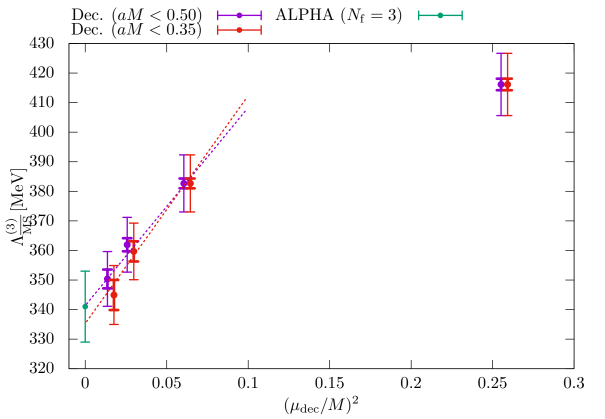

Figure 2: *Values for $\Lambda^{(3)}_{\overline{MS}}$ determined from the decoupling relation for three massive quarks and running in the pure gauge theory [6]. As $z = M/\mu_{dec}$ gets larger, the approximations for $\Lambda^{(3)}$ approach the results of [1]. The lines show possible extrapolations $M \to \infty$.*

strong coupling at large values of μ. The results are shown in Fig. 2. For $z = 4$ and higher, the behavior is well described by leading corrections to decoupling of order $O(M^{-2})$. In the $M \to \infty$ limit, a good agreement with the traditional determination [1], entirely in the $N_f = 3$ theory, is observed. At large z, where decoupling works best, the error is dominated by the difficult continuum extrapolation of the massive coupling. The other large contribution to the error is the uncertainty in μ_{dec}, which however will be significantly reduced once CLS updates their scale-setting [7,8].

In the extension of the project we plan to compute the massive coupling for $z = 6, 8$ on finer $L/a = 40, 48, 64$ lattices to improve on the continuum extrapolations shown in Fig. 1. Moreover to check the $M \to \infty$ extrapolation in Fig. 2 we want to simulate at $z = 12$ which is larger than the bottom quark mass ($z = 8$).

WWW

https://www.physik.hu-berlin.de

More Information

[1] M. Bruno, M. Dalla Brida, P. Fritzsch, T. Korzec, A. Ramos, S. Schaefer, H. Simma, S. Sint, R. Sommer, *Phys. Rev. Lett.* **119**, no. 10, 102001 (2017). doi:10.1103/PhysRevLett.119.102001

[2] M. Bruno, J. Finkenrath, F. Knechtli, B. Leder, R. Sommer, *Phys. Rev. Lett.* **114**, no. 10, 102001 (2015). doi:10.1103/PhysRevLett.114.102001

[3] F. Knechtli, T. Korzec, B. Leder, G. Moir, *Phys. Lett.* **B774**, 649 (2017). doi:10.1016/j.physletb.2017.10.025

[4] A. Athenodorou, J. Finkenrath, F. Knechtli, T. Korzec, B. Leder, M. Krstić Marinković, R. Sommer, *Nucl. Phys. B* **9**43 (2019) 114612. doi:10.1016/j.nuclphysb.2019.114612

[5] P. Fritzsch, A. Ramos, *JHEP* **1310**, 008 (2013). doi:10.1007/JHEP10(2013)008

[6] M. Dalla Brida and A. Ramos, *Eur. Phys. J. C* **79** (2019) no.8, 720. doi:10.1140/epjc/s10052-019-7228-z

[7] M. Bruno *et* al., *JHEP* **1502** (2015) 043. doi:10.1007/JHEP02(2015)043

[8] M. Bruno, T. Korzec and S. Schaefer, *Phys. Rev. D* **95** (2017) no.7, 074504. doi:10.1103/PhysRevD.95.074504

[9] M. Dalla Brida, R. Höllwieser, T. Korzec, F. Knechtli, A. Ramos and R. Sommer, *Phys. Lett.* **B807**, 649 (2020).

Project Partners

Bergische Universität Wuppertal, Trinity College Dublin, University Milan Bicocca