Measuring turbulence from within: a quasi-Lagrangian approach

Quasi-Lagrangian studies of spatio-temporal correlation in incompressible MHD turbulence

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In Short

- We run numerical studies of three-dimensional incompressible turbulence in neutral fluids and electrically conducting fluids (plasmas).
- Turbulent flows are composed of energetic structures of various sizes, which (in 3D turbulence) break up to ever smaller ones before being dissipated to heat at the smallest scales
- Our aim is to compute the correlation between velocity and magnetic field fluctuations on different length scales and with a time lag between them to deduce temporal and spatial properties of these nonlinear transfer processes
- We adopt the quasi-Lagrangian frame of reference to measure the fluctuations: tracer particles carry a set of probes at fixed distances, at which the velocity relative to the particle position is measured.

Turbulent flows are fundamental to various physical processes throughout the universe. On Earth, they can be observed in oceans, the atmosphere, smoke from a chimney or in the wake of an airplane. Examples in astrophysics include stellar winds, the generation of magnetic fields in celestial bodies, the propagation of cosmic rays and star formation from molecular clouds.

Turbulent flows can be viewed as being composed of vortices of various sizes which interact with each other in a complex, so-called “nonlinear” way, leading to chaotic, unpredictable behaviour. In three-dimensional turbulence, large vortices tend to break up into successively smaller structures, thereby transferring energy from large to small scales. At the smallest scales, the energy is converted into heat by viscous dissipation.

The underlying differential equations are the incompressible Navier-Stokes equations for neutral fluids and the magnetohydrodynamic (MHD) equations, which describe a plasma as a single electrically conducting fluid. Due to the complexity of the MHD equations, a comprehensive theory of MHD turbulence has not been formulated up to the present day. We aim at expanding our understanding of turbulence with direct numerical simulations (DNS), in which the governing equations are solved with a pseudo-spectral method.

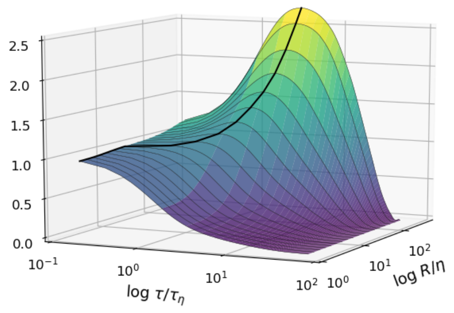

Figure 1: *Spatio-temporal correlation of fluctuations of varying length scale R and with small-scale fluctuations at a delayed time, determined by the time lag τ. The black line marks the time lag at which the correlation is maximal for a given R.*

The focus of this project is to compute spatio-temporal correlation functions of velocity and magnetic field fluctuations, to gain further insight into the properties of the nonlinear transfer processes. This was done before in the hydrodynamic case [1] and we aim at verifying those results at a higher resolution and extending this approach to MHD turbulence. The presence of a large-scale magnetic field makes this system anisotropic, i.e. the plasma behaves differently in the directions parallel and perpendicular to the magnetic field. We are particularly interested in the dynamics along these directions and in the influence of an external mean magnetic field.

During a simulation, we record a time series of the velocity and magnetic field fluctuations, from which the correlation between two fluctuations on certain length scales can be computed. By introducing a time lag between the time series of those two fluctuations, we can measure the temporal correlation. Due to the chaotic nature of turbulence, we expect the correlation to become zero as the time lag becomes comparable to the so-called “large-eddy turnover time”. This time scale estimates how long it takes for a large eddy to transfer its energy down to the dissipative scales and represents the longest time scale in the turbulent flow. Similarly, by varying the length scales of the fluctuations, we have access to the spatial correlation. Together, this information allows us to deduce transfer processes occuring on certain time and length scales. An example of this is shown in figure 1, in which the length scale of the velocity fluctuation at the delayed time is set to the smallest scale of the system. For a certain range of length scales, the correlation peaks at some finite time lag. This indicates that the large-scale fluctuations at a preceding time are correlated with the small-scale

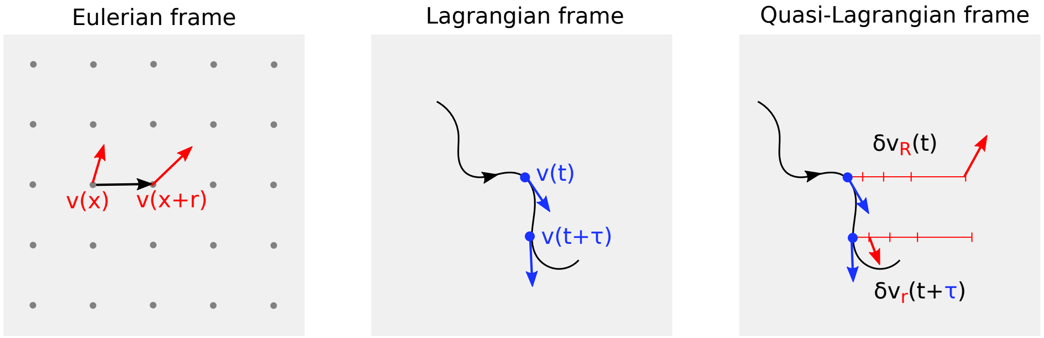

Figure 2: *Schematic figure of the Eulerian, Lagrangian and quasi-Lagrangian frames of reference.*

fluctuation at a delayed time, a manifestation of the direct energy cascade.

The fluctuations are obtained from numerical simulations, in which measurements typically are taken either in the standard Eulerian picture (fixed frame of reference) or the Lagrangian picture (co-moving with the fluid), as visualized in figure 2. While the former is suited for the computation of spatial correlations, the temporal aspect is not directly accessible due to the advection of small-scale structures by large-scale motions. In the Lagrangian frame of reference, trajectories of tracer particles are considered, which are suitable for the computation of temporal fluctuations but deliver only a subset of spatial information bound to the tracer trajectories. The so-called quasi-Lagrangian frame [2] combines both aspects in a controllable and well-defined approximation, by employing tracer particles which carry along with them a set of probes at fixed distances from the particles. The difference in velocity at the particle's position and the probe positions gives rise to the velocity fluctuations. The magnetic fluctuations are obtained similarly.

In short: by measuring the turbulence from within, we aim at gaining further insights into the properties of the nonlinear transfer processes to help building a comprehensive theory of MHD turbulence.

WWW

`https://www-astro.physik.tu-berlin.de/en/node/343`

More Information

[1] L. Biferale, E. Calzavarini, F. Toschi, *Physics of Fluids* **23.8**, 085107 (2011) doi: 10.1063/1.3623466

[2] V. I. Belinicher, V.S. L'vov. *Sov. Phys. JETP* **66.2**, 303-313 (1987).

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