

The butterfly effect in turbulence

Lyapunov exponents and turbulent transport in hydrodynamic and magnetohydrodynamic turbulence

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In Short

- Most astrophysical flows are highly turbulent and characterized by rapidly varying multiscale motions.
- Turbulent flows are also almost always chaotic such that small changes in the initial conditions lead to exponential divergence of solutions. This is often referred to as the butterfly effect.
- The timescales describing such diverging solutions are quantified by Lyapunov exponents. The analysis of the Lyapunov exponents is required for a more fundamental theoretical understanding of turbulence.
- Turbulence is also efficient at smoothing large-scale structures. Such turbulent diffusion is important in many astrophysical phenomena.
- We will compute Lyapunov exponents and turbulent diffusion of flows and temperature from state-of-the-art simulations of turbulence with and without magnetic fields.

Introduction. Almost all of the flows in the Earth's atmosphere, the interplanetary space, solar and stellar convection zones, and in other astrophysical systems are turbulent. This means that the flows change rapidly and in chaotic fashion on a huge number of scales from the system size to microscopic scales. It is important to understand such turbulent flows which play important roles in numerous contexts such as engineering, the climate of the Earth, and the interiors of stars.

Research questions. We will measure the Lyapunov exponents of the velocity field from high resolution simulations of turbulence reaching significantly more turbulent parameter regimes than previously in the literature. Such simulations are needed to shed light on fundamental questions regarding turbulent flows and their theoretical interpretation. A number of theoretical ideas were recently presented in a theoretical study [1] which forms the basis for the numerical simulations.

Furthermore, we measure the turbulent diffusion of velocity and temperature from three dimensional simulations of turbulence. This part of the project is a follow-up to [2] where turbulent viscosity and

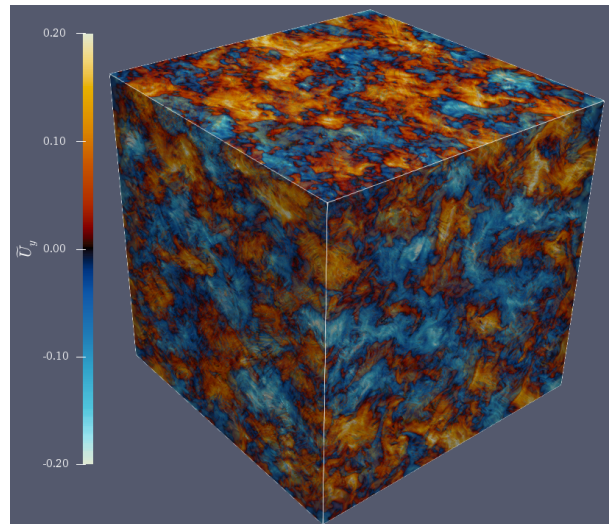


Figure 1: Streamwise velocity U_y in a turbulence simulation with imposed shear flow. Adapted from [2].

magnetic diffusion where measured. Of particular interest in the current project are the effects of magnetic fields on the turbulent diffusion which has implications for various phenomena in stellar convection zones.

Models and methods. We use the Pencil Code [3,4] which is a free (licensed under GNU GPL v3) simulation framework for solving a great variety of physical problems ranging from gravitational waves, chemistry, planet formation to stellar magnetism. We use a simplified cubic geometry where turbulence is driven by external forcing. This approach allows the minimal ingredients necessary to the problem at hand to be included with maximal control. Figure [1] shows a representative example of the turbulence simulations.

WWW

<https://www.uni-goettingen.de/en/203293.html>

More Information

- [1] I. Fouxon, J. Feinberg, P.J. Käpylä, M. Mond, *Phys. Rev. E* **103**, 033110 (2021). doi: 10.1103/PhysRevE.103.033110
- [2] P.J. Käpylä, M. Rheinhardt, A. Brandenburg, M.J. Käpylä, *Astron. Astrophys.* **636**, 93 (2020). doi:10.1051/0004-6361/201935012

[3] The Pencil Code Collaboration, *Journal of Open Source Software* **6**, 2807 (2021). doi: 10.21105/joss.02807

[4] Github: <https://github.com/pencil-code>
Nordita: <http://pencil-code.nordita.org/>

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